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$B \to \pi$ and $B \to K$ transitions from QCD sum rules on the light-cone

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Abstract: I calculate the form factors describing semileptonic and penguin-induced decays of B mesons into light pseudoscalar mesons. Theform factors are calculated from QCD sum rules on the light-cone including contributions up to twist 4, radiative corrections to the leading twist contribution and SU(3)-breaking effects. The theoretical uncertainty is estimated to be $\sim 20\%$. The heavy-quark-limit relations between semileptonic and penguin form factors are found to be valid in the full accessible range of momentum transfer.

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Decays of B mesons into light mesons offer the possibility to access the less well known entries in the CKM quark mixing matrix such as V_{ub} and V_{ts}. The measurement of rare penguin-induced B decays may also give hints at new physics in the form of loop-induced effects. With new data of hitherto unknown precision from the new experimental facilities BaBar at SLAC and Belle at KEK expected to be available in the near future, the demands for accuracy of theoretical predictions are ever increasing. The central problem of all such predictions, our failure to solve non-perturbative QCD, is well known and so far prevents a rigorous calculation of form factors from first principles. Theorists thus concentrate on providing various approximations. The maybe most prominent of these, simulations of QCD on the lattice, have experienced considerable progress over recent years; the current status for B decays is summarized in [1]. It seems, however, unlikely that lattice calculations will soon overcome their main restriction in describing $b \to u$ and $b \to s$ transitions, namely the effective upper cut-off that the finite lattice spacing imposes on the momentum of the final-state meson. The cut-off restricts lattice predictions of B decay form factors to rather large momentum transfer q^2 of about $15\,\mathrm{GeV}^2$ or larger. The physical range in B decays, however, extends from 0 to about $20\,\mathrm{GeV}^2$, depending on the process; for radiative decays like $B \to K^*\gamma$ it is exactly $0\,\mathrm{GeV}^2$. Still, one may hope to extract from the lattice data some information on form factors in the full physical range, as their behaviour at large q^2 restricts the shape at small q^2 via the analytical properties of a properly chosen vacuum correlation function. The latter function, however, also contains poles and multi-particle cuts whose exact behaviour is not known, which limits the accuracy of bounds obtained from such unitarity constraints and until now has restricted their application to $B \to \pi$ transitions [2, 3]. The most optimistic overall theoretical uncertainty one may hope to obtain from this method is the one induced by the input lattice results at large q^2, which to date is around (15–20)% [4, 2]. A more model-dependent extension of the lattice form factors into the low q^2 region is discussed in [5].

An alternative approach to heavy-to-light transitions is offered by QCD sum rules on the light-cone. In contrast and complementary to lattice simulations, it is just the fact that the final-state meson *does* have large energy and momentum of order $\sim m_B/2$ in a large portion of phase-space that is used as starting point (which restricts the method to not too large momentum transfer, to be quantified below). The key-idea is to consider $b \to u$ and $b \to s$ transitions as hard exclusive QCD processes and to combine the well-developed description of such processes in terms of perturbative amplitudes and non-perturbative hadronic distribution amplitudes [6] (see also [7] for a nice introduction) with the method of QCD sum rules [8] to describe the decaying hadron. The idea of such "light-cone sum rules" was first formulated and carried out in [9] in a different context for the process $\Sigma \to p\gamma$, and its first application to B decays was given in [10]. Subsequently, light-cone sum rules were considered for many B decay processes, see [11, 12] for reviews. As light-cone sum rules are based on the light-cone expansion of a correlation function, they can be systematically improved by including higher twist contributions and radiative corrections to perturbative amplitudes. The first calculation in [10] was done at tree-level and to leading twist 2 accuracy. In [13, 14], twist 3 and 4 contributions were included, and in [15], one-loop radiative corrections to the twist 2 contribution to the form factor f_+^π were calculated. In [16], the corresponding radiative corrections to the decays of B mesons into the vector mesons ρ, K^*, ϕ were calculated. In [17], the scalar form factor f_0^π was calculated at tree-level. In the present letter, I calculate the remaining radiative corrections to all semileptonic and penguin $B \to \pi$ and $B \to K$ transitions and present new and more accurate results for the corresponding form factors.

Let me begin by defining the relevant quantities. The semileptonic form factors are defined as ($q = p_B - p$)

$$\langle P(p)|\bar{q}\gamma_\mu b|B(p_B)\rangle = f_+^P(q^2)\left\{(p_B+p)_\mu - \frac{m_B^2-m_P^2}{q^2}\,q_\mu\right\} + \frac{m_B^2-m_P^2}{q^2}\,f_0^P(q^2)\,q_\mu, \quad (1)$$

where P stands for the pseudoscalar meson π or K and $q = u$ for the π and $q = s$ for the K. The penguin form factor is defined as

$$\begin{aligned}\langle P(p)|\bar{q}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_B)\rangle &\equiv \langle P(p)|\bar{s}\sigma_{\mu\nu}q^\nu b|B(p_B)\rangle \\ &= i\left\{(p_B+p)_\mu q^2 - q_\mu(m_B^2-m_K^2)\right\}\frac{f_T^P(q^2)}{m_B+m_K}. \quad (2)\end{aligned}$$

The physical range in q^2 is $0 \leq q^2 \leq (m_B - m_P)^2$. Although there are of course no semileptonic decays $B \to Ke\nu$, the above form factors contribute to, say, $B \to K\ell\bar{\ell}$. Recalling the results of perturbative QCD for the π electromagnetic form factor as summarized in [7], one may suppose that the dominant contribution to the above form factors be the exchange of a hard perturbative gluon between, for instance, the u quark and the antiquark; this possibility was advocated in [18]. This is, however, not the case, and it was pointed out already in Ref. [10] that the dominant contribution comes from the so-called Feynman mechanism, where the quark created in the weak decay carries nearly all of the final-state meson's momentum, while all other quarks are soft, and which bears no perturbative suppression by factors α_s/π. In an expansion in the inverse b quark mass, the contribution from the Feynman mechanism is of the same order as the gluon-exchange contribution with momentum fraction of the quark of order $1 - \Lambda_{\rm QCD}/m_b$, but it dies off in the strict limit $m_b \to \infty$ due to Sudakov effects. This means that — unlike the case of the electromagnetic π form factor — knowledge of the hadron distribution amplitudes

$$\phi(u,\mu^2) \sim \int_0^{\mu^2} dk_\perp^2\,\Psi(u,k_\perp),$$

where Ψ is the full Fock-state wave function of the B and $\pi(K)$, respectively, u is the longitudinal momentum fraction carried by the (b or $u(s)$) quark, $k_\perp$ is the transverse quark momentum, is not sufficient to calculate the form factors in the form of overlap integrals

$$F \sim \int_0^1 du\,dv\,\phi^*_{\pi(K)}(u)\,T_{\rm hard}(u,v;q^2)\,\phi_B(v)$$

(with $T_{\rm hard}\propto\alpha_s$).[1] Instead, in the method of light-cone sum rules, only the light meson is described by distribution amplitudes. Logarithms in $k_\perp$ are taken into account by the evolution of the distribution amplitudes under changes in scale, powers in $k_\perp$ are taken into account by higher twist distribution amplitudes. The B meson, on the other hand, is described, as in QCD sum rules, by the pseudoscalar current $\bar{d}i\gamma_5 b$ in the unphysical region with virtuality $p_B^2-m_b^2\sim O(m_b)$, where it can be treated perturbatively. The real B meson, residing on the physical cut at $p_B^2=m_B^2$, is then traced by analytical continuation, supplemented by the standard QCD sum rule tools to enhance its contribution with respect to that of higher single- or multi-particle states coupling to the same current.

The starting point for the calculation of the form factors in (1) and (2) is thus the correlation functions ($j_B=\bar{d}i\gamma_5 b$):

$$\mathrm{CF}_V = i\int d^4y e^{iqy}\langle P(p)|T[\bar{q}\gamma_\mu b](y)j_B^\dagger(0)|0\rangle \;=\; \Pi_+^P(q+2p)_\mu+\Pi_-^P q_\mu, \tag{3}$$

$$\mathrm{CF}_T = i\int d^4y e^{iqy}\langle P(p)|T[\bar{q}\sigma_{\mu\nu}q^\nu b](y)j_B^\dagger(x)|0\rangle \;=\; 2iF_T^P(p_\mu q^2-(pq)q_\mu), \tag{4}$$

which are calculated in an expansion around the light-cone $x^2=0$. The expansion goes in inverse powers of the b quark virtuality, which, in order for the light-cone expansion to be applicable, must be of order m_b. This restricts the accessible range in q^2 to $m_b^2-q^2 \stackrel{<}{\sim} O(m_b)$ parametrically. For physical B mesons, I choose $m_b^2-q^2\leq 17\,\mathrm{GeV}^2$. Note also that for very large q^2 the influence of the next nearby pole (B^* for f_+^π) becomes more prominent.

It proves convenient to perform the calculation for an arbitrary weak vertex $\Gamma=\{\gamma_\mu,\ \sigma_{\mu\nu}q^\nu\}$, which, neglecting for the moment radiative corrections, yields:

$$\begin{aligned}
\mathrm{CF}_\Gamma = {} & \frac{f_\pi}{4}\int_0^1 du\Bigg[-\phi_\pi(u)Tr(\Gamma S_b(Q)\widehat{p})+\\
& +\frac{m_\pi^2}{m_u+m_d}\left\{-\phi_P(u)Tr(\Gamma S_b(Q))+\frac{i}{6}\,\phi_\sigma(u)\,\frac{\partial}{\partial Q_\alpha}\,Tr(\Gamma S_b(Q)\sigma_{\alpha\beta})p^\beta\right\}\\
& +\left\{g_1(u)-\int_0^u dv\, g_2(v)\right\}\frac{\partial^2}{\partial Q_\alpha\partial Q^\alpha}\,Tr(\Gamma S_b(Q)\widehat{p})-g_2(u)\,\frac{\partial}{\partial Q_\alpha}\,Tr(\Gamma S_b(Q)\gamma_\alpha)\Bigg]\\
& +\frac{f_\pi}{4}\int_0^1 dv\int_0^1\mathcal{D}\underline{\alpha}\,\frac{1}{\widetilde{s}^2}\Bigg[\frac{4f_{3\pi}}{f_\pi}\,v(pq)\phi_{3\pi}(\underline{\alpha})Tr(\Gamma\widehat{p})+(2\phi_\perp(\underline{\alpha})-\phi_\|(\underline{\alpha}))Tr(\Gamma(\widehat{q}+m_b)\widehat{p})\\
& \qquad +2v\left\{\phi_\|(\underline{\alpha})Tr(\Gamma\widehat{p}\widehat{q})-2(pq)\phi_\perp(\underline{\alpha})Tr(\Gamma)\right\}+\left\{2\widetilde{\phi}_\perp(\underline{\alpha})-\right.\\
& \qquad \left.-\widetilde{\phi}_\|(\underline{\alpha})\right\}\,Tr(\Gamma(\widehat{q}+m_b)\widehat{p})+4iv\widetilde{\phi}_\perp(\underline{\alpha})Tr(\Gamma\sigma_{\alpha\beta})q^\alpha p^\beta\Bigg].
\end{aligned}\tag{5}$$

[1]Note also that not much is known about ϕ_B, whereas the analysis of light meson distribution amplitudes is facilitated by the fact that it can be organized in an expansion in conformal spin, much like the partial wave expansion of scattering amplitudes in quantum mechanics in rotational spin.

Explicit expressions for $\Pi_\pm$ and F_T were already obtained in [13, 14]. Here $Q = q + \bar{u}p$, $s = m_b^2 - Q^2 = m_b^2 - up_B^2 - \bar{u}q^2$, $\mathcal{D}\underline{\alpha} = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$ and $\widetilde{s} = m_b^2 - (q + (\alpha_1 + v\alpha_3)p)^2$; $S_b(Q) = (\widehat{Q} + m_b)/(-s)$ is the b quark propagator. In the above expression, $\phi_{\pi,K}$ is the leading twist 2 distribution amplitude, ϕ_P and ϕ_σ are the two-particle distribution amplitudes of twist 3, g_1 and g_2 those of twist 4, all of which are defined in [19]. The twist 3 and 4 two-particle distribution amplitudes are determined completely in terms of the twist 3 and 4 three-particle distribution amplitudes $\phi_{3\pi}$, $\phi_{\|,\perp}$ and $\widetilde{\phi}_{\|,\perp}$ [19]. Note that in the above expression corrections in the light meson mass are neglected ($m_\pi^2/(m_u + m_d)$, however, is expressed in terms of the quark condensate and taken into account; this also removes any strong dependence on the rather poorly known light quark masses). Their inclusion, of potential relevance in $B \to K$ transitions, is not straightforward and requires an extension of the method developed in Ref. [19] to include meson- and quark-mass corrections in the twist 4 distribution amplitudes. According to [20], the numerical impact on the form factors is small, around 5%, and most pronounced at large q^2.

It is convenient to calculate also the radiative corrections for arbitrary weak vertex. To twist 2 accuracy, the light quarks are massless and carry only longitudinal momentum. The one-loop calculation does not occasion any particular technical complications, but results in bulky expressions which I refrain from quoting here.[2] The general structure is, as to be expected, similar to that for the form factor f_+^π obtained in [15]. The separation of perturbative and non-perturbative contributions introduces an arbitrary logarithmic (infra-red) factorization scale. The condition that the correlation function be independent of that scale leads to an evolution equation for the distribution amplitude, which was first derived and solved in [6] to leading logarithmic accuracy. In the present context, with full $O(\alpha_s)$ corrections to the perturbative part included, one has to use the next-to-leading order evolution of the distribution amplitude, which was derived in closed form in [21]. A natural choice for the factorization scale is the virtuality of the b quark, $\mu_{\rm IR}^2 \sim u\, m_b$. For technical reasons it is, however, more convenient to choose a fixed scale like $\mu_{\rm IR}^2 = m_B^2 - m_b^2$, which is of the same order. The numerical impact of changing the scale is minimal.[3] The penguin form factor depends also on an ultra-violet scale, the renormalization-scale of the local operator $\bar{q}\sigma_{\mu\nu}q^\nu b$ appearing in the effective weak Lagrangian. A natural choice for this ultra-violet scale is $\mu_{\rm UV} = m_b$.

As for the radiative corrections, it turns out that they are dominated by the correction to the pseudoscalar B vertex, which, as discussed below, yields large cancellations against the corresponding corrections to the leptonic B decay constant f_B.

Let me now derive the light-cone sum rules. The correlation functions CF_Γ, calculated for unphysical p_B^2, can also be written as dispersion relations over the physical

[2]The expressions were generated by a MATHEMATICA-program which, together with the results, can be obtained from the author upon request.

[3]This is in contrast to the π electromagnetic form factor, which is rather sensitive to the shape of the distribution amplitude near the end-points.

cut. Singling out the contribution of the B meson, one has for instance for Π_+:

$$\mathrm{CF}_{\Pi_+} = \frac{m_B^2 f_B}{m_b}\, f_+(q^2)\, \frac{1}{m_B^2 - p_B^2} + \text{higher poles and cuts}, \tag{6}$$

where f_B is the leptonic decay constant of the B meson, $f_B m_B^2 = m_b\langle B|j_B^\dagger|0\rangle$. In order to enhance the ground-state B contribution to the right-hand side, I perform a Borel-transformation:

$$\hat{B}\,\frac{1}{s-p_B^2} = \frac{1}{M^2}\exp(-s/M^2), \tag{7}$$

with the Borel parameter M^2. The next step is to invoke quark-hadron duality to approximate the contributions of hadrons other than the ground-state B meson, so that finally

$$\hat{B}\,\mathrm{CF}_{\Pi_+} = \frac{1}{M^2}\,\frac{m_B^2 f_B}{m_b}\, f_+(q^2)\, e^{-m_B^2/M^2} + \frac{1}{M^2}\,\frac{1}{\pi}\int_{s_0}^\infty ds\, \mathrm{Im}\mathrm{CF}_{\Pi_+}(s)\, \exp(-s/M^2). \tag{8}$$

This equation is the light-cone sum rule for f_+; those for f_0 and f_T look similar. Here s_0 is the so-called continuum threshold, which separates the ground-state from the continuum contributions; s_0 and M^2 are in principle free parameters of the light-cone sum rules. As for s_0, it turns out that eq. (8) and the corresponding relations for the other form factors are largely independent of it (e.g. changing s_0 within 30 and 100 GeV^2 changes the result by only 10 to 15%, depending on q^2). It is thus appropriate to fix s_0 from the QCD sum rule for f_B, by which one has to divide (8) in order to obtain the form factor; from the analysis of the sum rule for f_B, one finds $s_0 \approx 34\,\mathrm{GeV}^2$. As for M^2, on the other hand, the QCD sum rule for f_B is nearly independent of it, changing by only 2% within the optimized range $M^2 \approx$(4–8) GeV^2 in which the sum rule is stable (and yielding values for f_B of (150–200) MeV, in perfect agreement with the results from lattice simulations). It is thus appropriate to use the same values of the Borel parameter both in (8) and the sum rules for f_B, $M^2 \approx$(4–8) GeV^2, and to include the remaining $\pm 10\%$ on it in the total theoretical uncertainty. Note also that subtraction of the continuum contribution from both sides of (8) introduces a lower limit of integration $u \geq (m_b^2 - q^2)/(s_0 - q^2)$ in (5), which behaves as $1 - \Lambda_{\mathrm{QCD}}/m_b$ for large m_b and thus corresponds to the dynamical configuration of the Feynman mechanism.

Let me now specify the non-perturbative input. For the b quark I use the one-loop pole mass $m_b = (4.8 \pm 0.1)\,\mathrm{GeV}$, which is consistent with a recent determination from the Υ mesons [22]. For the light mesons, the distribution amplitudes need to be specified. Fortunately, conformal symmetry of massless QCD combined with the nonlocal string operators technique developed in [23], provides a very powerful tool to describe higher twist distribution amplitudes in a mutually consistent and most economic way (see [24] for a detailed discussion). The determination of the relevant non-perturbative parameters from QCD sum rules was pioneered in [25]. In [19], the twist 3 and 4 π distribution amplitudes were obtained including contributions up to conformal spin 11/2 in terms of 6 independent non-perturbative parameters whose values were determined from QCD sum rules. The leading twist 2 distribution amplitude, on the other hand, can be expanded in Gegenbauer polynomials $C_i^{3/2}$:

$$\phi_{\pi,K} = 6u(1-u)\left(1+\sum_{i=1}^{\infty} a_i(\mu)C_i^{3/2}(2u-1)\right). \tag{9}$$

The Gegenbauer moments a_i renormalize multiplicatively. For π, all odd moments vanish because of the π's definite G-parity. In practice, one truncates the expansion after the first few terms,

$$\phi_{\pi,K(n)} = 6u(1-u)\left(1+\sum_{i=1}^{n} a_i(\mu)C_i^{3/2}(2u-1)\right),$$

with $n=4$ (for π) or $n=2$ (for K). I will discuss the impact of this truncation on the form factors later. For now, I use the π distribution amplitude as obtained in [26] (see also [27]),

$$a_2^\pi(1\,\mathrm{GeV}) = 0.44, \quad a_4^\pi(1\,\mathrm{GeV}) = 0.25. \tag{10}$$

For the K, on the other hand, the non-zero value of the strange quark mass induces non-vanishing values of the odd moments. I use

$$a_1^K(1\,\mathrm{GeV}) = 0.17, \quad a_2^K(1\,\mathrm{GeV}) = 0.2, \tag{11}$$

where the first value was obtained in [25] and the second one comes from an analysis of the sum rule for the π in [26], due account being taken of SU(3)-breaking effects.

The results are displayed in fig. 1. The form factor f_+^π coincides with the one obtained in [15]. I plot each form factor using the twist 2 distribution amplitudes as specified above and with and without $O(\alpha_s)$ corrections, and also using the asymptotic distribution amplitudes $\phi_{\pi,(0)}$ and $\phi_{K,(1)}$ to illustrate the impact of non-asymptotic contributions. The plotted curves were obtained with $m_b = 4.8\,\mathrm{GeV}$, $s_0 = 33.5\,\mathrm{GeV}^2$ and $M^2 = 6\,\mathrm{GeV}^2$. The distribution amplitudes are evaluated at the scale $\mu^2 = m_B^2 - m_b^2 =: \mu_b^2$. Apparently, the net effect of radiative corrections on the form factors is rather small. This is due to an effect already observed in [15]: the radiative corrections to the QCD sum rule for f_B are rather large, which is due to the large vertex corrections to the pseudoscalar B vertex. In the radiative corrections to the light-cone sum rules, the same vertex appears with corrections of similar size, so that they cancel between left- and right-hand side of (8), leaving a net effect of around 10%.

For all form factors, the effect of three-particle twist 3 and 4 quark-gluon contributions (and their induced effects in the two-particle distribution amplitudes) are small ($\sim 5\%$), so that the considerable theoretical uncertainty of these terms does not play. This also shows that the expansion in contributions of increasing twist is under good control. The remaining twist 3 contributions are proportional to the quark condensate, which, as already noted in [15], introduces only a small uncertainty in the final results.

As is expected from the definition of f_0, which refers to a scalar current, it increases less sharply in q^2 than the other form factors. A good parametrization for the q^2 dependence can be given in terms of three parameters as

$$F(q^2) = \frac{F(0)}{1 - a_F\,(q^2/m_B^2) + b_F\,(q^2/m_B^2)^2}. \tag{12}$$

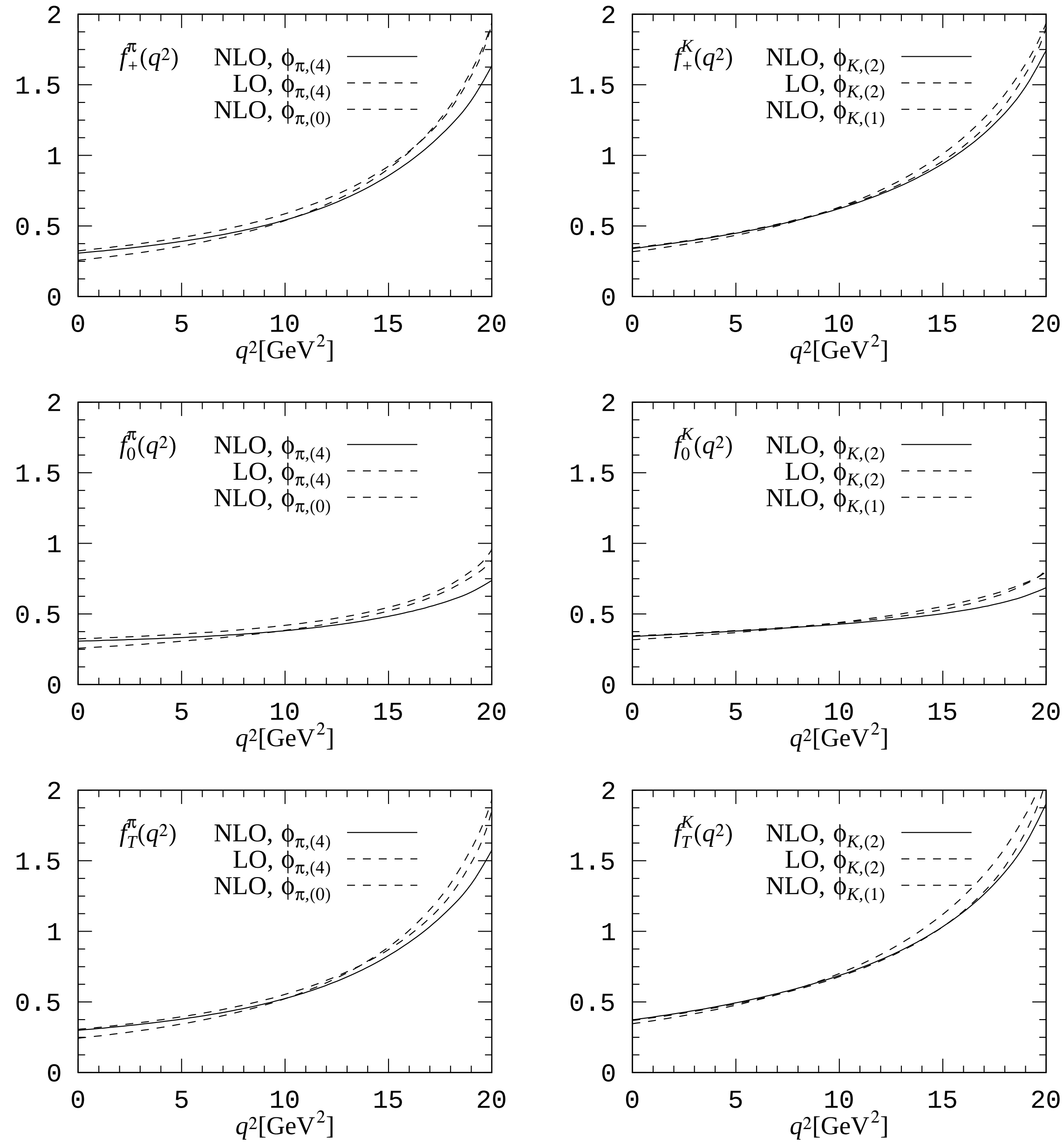

Figure 1: Form factors from light-cone sum rules in various approximations.

The parameters are given in Table 1 for central values of the input parameters. For comparison, I also give the results for f_+^π quoted in [12] and f_+^K obtained in [13], the latter being obtained in leading-logarithmic accuracy. The table confirms what can also be inferred from the figure, namely that, for both π and K, mesons f_+ and f_T nearly coincide. Comparison with the K form factors shows that the main SU(3)-breaking effect is in the normalization $F(0)$, whereas the q^2 dependence is only slightly modified. This can be understood from the fact that the formation of a π or K meson is proportional to their respective decay constants $f_{\pi,K}$, so that one would naively expect an enhancement $\sim f_K/f_\pi = 1.2$ of the K form factors (at least if the three-parton states are not important), which is essentially what I find.

	f_+^π	f_+^K	f_0^π	f_0^K	$f_T^\pi(m_b)$	$f_T^K(m_b)$	f_+^π [12]	$f_+^{K,LO}$ [13]
$F(0)$	0.305	0.341	$\equiv f_+^\pi(0)$	$\equiv f_+^K(0)$	0.296	0.374	0.27 ± 0.05	0.33 ± 0.05
a_F	1.29	1.41	0.266	0.410	1.28	1.42	1.50	1.14
b_F	0.206	0.406	-0.752	-0.361	0.193	0.434	0.52	0.05

Table 1: Results for form factors with $m_b = 4.8\,\mathrm{GeV}$, $s_0 = 33.5\,\mathrm{GeV}^2$ and $M^2 = 6\,\mathrm{GeV}^2$ in the parametrization of eq. (12). Renormalization scale for f_T is $\mu = m_b$. The theoretical uncertainty is $\sim 20\,\%$.

q^2	$f^\pi_{+,\mathrm{latt}}(q^2)$[4, 2]	$f^\pi_{+,\mathrm{LCSR}}(q^2)$	$f^\pi_{0,\mathrm{latt}}(q^2)$[4, 2]	$f^\pi_{0,\mathrm{LCSR}}(q^2)$
$14.9\,\mathrm{GeV}^2$	0.85 ± 0.20	0.85 ± 0.15	0.46 ± 0.10	0.5 ± 0.1
$17.2\,\mathrm{GeV}^2$	1.10 ± 0.27	1.1 ± 0.2	0.49 ± 0.10	0.55 ± 0.15
$20.0\,\mathrm{GeV}^2$	1.72 ± 0.50	1.6	0.56 ± 0.12	0.7

Table 2: Comparison of lattice results for $B \to \pi$ form factors with results from light-cone sum rules. The errors for lattice results are those quoted in [2].

Varying all input parameters within their respective allowed ranges, I obtain uncertainties between 10 and 15%. Combining this with the systematic uncertainty ~10% introduced by the need to separate the ground-state B contribution from that of higher states, i.e. the uncertainty introduced by quark-hadron duality, the final uncertainty of the form factors is ~20%. A slight reduction of this uncertainty may be possible if more accurate information on the twist 2 distribution amplitude becomes available, for instance from lattice simulations, but it is in principle impossible to reduce the uncertainty from using quark-hadron duality to isolate the B meson, which thus limits the total accuracy of the method of light-cone sum rules. A calculation of radiative corrections to the twist 3 contributions is in principle feasible, although complicated, but will hardly reduce the uncertainty which is dominated by the one of the quark condensate.

A comparison with lattice results from the UKQCD collaboration is given in Table 2. The agreement with the lattice data is excellent, as it was also found for $B \to \rho, K^*$ form factors in [16]. The LCSR point at $q^2 = 20\,\mathrm{GeV}^2$ is just for illustration, because of which I also refrain from assigning it an error.

In the limit $m_b \to \infty$, Isgur and Wise have obtained a relation between the semileptonic and the penguin form factors [28]:

$$\frac{f_T}{m_B + m_P} = \frac{1}{2m_b}\left\{\left(1 + \frac{m_b^2 - q^2}{q^2}\right) f_+ - \frac{m_b^2 - q^2}{q^2}\, f_0\right\}, \tag{13}$$

which is strictly valid only near zero recoil (i.e. near $q^2 = m_b^2$). In Fig. 2 I plot the left- and right-hand sides of eq. (13) over the full range of q^2. The agreement between the curves for *all* q^2 is striking; they differ by 3% at $q^2 = 0$ and by less than 1% at $q^2 = 17\,\mathrm{GeV}^2$. A closer inspection of the underlying light-cone sum rules shows

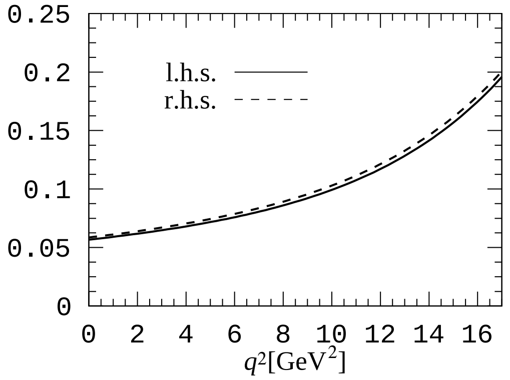

Figure 2: Isgur-Wise relation (13) between penguin and semileptonic form factors.

that to twist 2 accuracy (13) is valid exactly and for arbitrary m_b already at the level of correlation functions $\Pi_\pm$ and F_T and thus — to that accuracy — is independent of the details of the extraction of the B meson contribution. For the twist 3 and 4 contributions to $\Pi_\pm$ and F_T, (13) holds in the kinematical regime characteristic for the Feynman mechanism, i.e. near $u \sim 1$, and up to terms which are suppressed by one power of m_b, which account for the 3% breaking of (13) by the light-cone sum rule results.

As the uncertainty associated with the twist 3 two-particle distribution amplitudes and the higher twist amplitudes is small, and the form factor thus depends essentially on the quality of information on the twist 2 distribution amplitude, it is worthwhile to investigate in more detail the impact of truncating the series (9) after the first few terms. Let me first recall that in [26] $a_2^\pi(1\,\mathrm{GeV})$ was determined from a QCD sum rule for Gegenbauer moments of in principle arbitrary degree, whereas a_4^π was obtained from requiring $\phi_\pi(1/2, 1\,\mathrm{GeV}) = 1.2 \pm 0.3$, which follows from a QCD sum rule for the πNN-coupling. The reason for not considering QCD sum rules for higher moments is that they show a strong divergence with the degree n, rendering the series in (9) highly divergent. In order to simulate possible effects of higher moments without distorting the asymptotic $u(1-u)$ behaviour near the end-points too much, I allow for a logarithmic divergence of the sum in Gegenbauer polynomials, yielding the following models:

$$\begin{aligned}
\text{model I}: \quad & \phi_\pi(1/2, \mu = 1\,\mathrm{GeV}) \;=\; 1.2,\; a_2^\pi \text{ model-dependent:}\\
\phi_\pi^I(u, \mu = \mu_b) &= 6u(1-u) - 0.95 \cdot 6u(1-u)\,\frac{3}{5}\,\ln u \ln(1-u), \qquad (14)\\
\text{model II}: \quad & \phi_\pi(1/2, \mu = 1\,\mathrm{GeV}) \;=\; 1.2,\; a_2(1\,\mathrm{GeV}) = 0.44:\\
\phi_\pi^{II}(u, \mu = \mu_b) &= 6u(1-u)\{1 + 0.35\, C_2^{3/2}(2u-1)\}\\
& \quad - 8.5 \cdot 6u(1-u)\left\{\frac{3}{5}\,\ln u \ln(1-u) + 1 + \frac{7}{50}\,C_2^{3/2}(2u-1)\right\}. \qquad (15)
\end{aligned}$$

The corresponding Gegenbauer-spectra fall off as $1/n^3$:

$$\begin{aligned}
(\mathrm{I}):\; \{a_n\} &= \{1, 0.13, 0.030, 0.011, \ldots\}\\
(\mathrm{II}):\; \{a_n\} &= \{1, 0.35, 0.27, 0.10, 0.049, 0.027, \ldots\}.
\end{aligned}$$

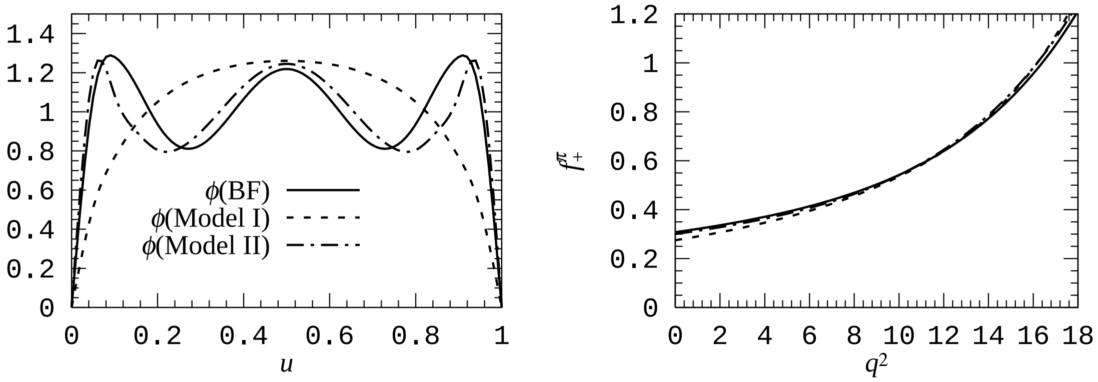

Figure 3: Dependence of f_+^π on the twist 2 π distribution amplitude.

In fig. 3 I plot the above model distribution amplitudes as well as the one suggested by Braun and Filyanov [26], i.e. $\phi_{\pi(4)}$. Although they look rather different, the resulting form factors f_+^π, also shown in Fig. 3, vary by at most 10%, i.e. are within the theoretical error. It is evident that the form factors do not depend on the details of the Gegenbauer-spectrum, but are sensitive only to a few gross characteristics like the value of the distribution amplitude at one point (different from the end-points) and the first one or two moments. This is true as long as the distribution amplitudes are folded with smooth functions (as it is the case for form factors), so that higher order oscillatory Gegenbauer polynomials are effectively "washed out". A determination of the relevant few characteristics from an independent source, e.g. lattice simulations, would evidently help to further increase the accuracy of form factors calculated from light-cone sum rules.

Summarizing, I have calculated the semileptonic and penguin form factors of $B \to \pi$ and $B \to K$ transitions from light-cone sum rules. A new feature was the inclusion of one-loop radiative corrections to the leading twist contributions. The results are summarized in Fig. 1 and Table 1. The impact of radiative corrections and higher twist contributions is small, so that the achievable accuracy is limited by the inherent systematic uncertainty of light-cone sum rules, which is associated with the extraction of the B meson ground-state contribution out of the continuum of states coupling to the same current. This uncertainty is estimated to be $\sim 10\%$ and of the same size as the uncertainty induced by the input parameters in the sum rule. Hence, further refinement of the calculation by including higher twist contributions or two-loop radiative corrections is not expected to yield higher accuracy of the result. It would, however, be useful to have an independent determination of the few lowest moments of the twist 2 π and K meson distribution amplitudes from lattice simulations. The existing results [29] have large uncertainties, and in view of the recent improvements of the methods of lattice QCD and the availability of much more powerful computers, more accurate results seem within reach. Very recently [30], a new method was suggested to calculate the leading twist distribution amplitude on the lattice directly as a function of u. If feasible with statistical and systematic errors in the 20% range, this would help to reduce the total uncertainty of the $B \to \pi, K$ form factors to $\sim 15\%$. The application of these lattice results would not be restricted to B meson decays, but also of direct relevance to the description of other hard exclusive processes, for instance single-meson production at HERA.

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